Area Under The Curves

Question1

The area of the region (in sq units) bounded by the curves $x^2 + y^2 = 16$ and $y^2 = 6x$ is

AP EAPCET 2025 - 26th May Evening Shift

Options:

A.

$4\pi + 4\sqrt{3}$

B.

$\frac{2}{3}(4\pi + \sqrt{3})$

C.

$\frac{4}{3}(4\pi + \sqrt{3})$

D.

$\frac{4\pi+\sqrt{3}}{3}$

Answer: C

Solution:

Given:

The circle is $x^2 + y^2 = 16$.

The parabola is $y^2 = 6x$.

Finding Intersection Points:

To find where these curves meet, put y^2 from the parabola into the circle's equation:

$$
\begin{aligned}
\rightarrow \quad & x^2+y^2=16 \\
& x^2+6x=16 \\
& x^2+6x-16=0 \\
& (x+8)(x-2)=0 \\
& x=-8,\ 2
\end{aligned}
$$

We only use $x=2$ (since $x=-8$ is outside the region for $y^2=6x$ and gives no real y for $x^2+y^2=16$).

For $x=2$, $y^2=6\cdot 2=12$. So $y=\sqrt{12}$ and $y=-\sqrt{12}$.

Finding the Area:

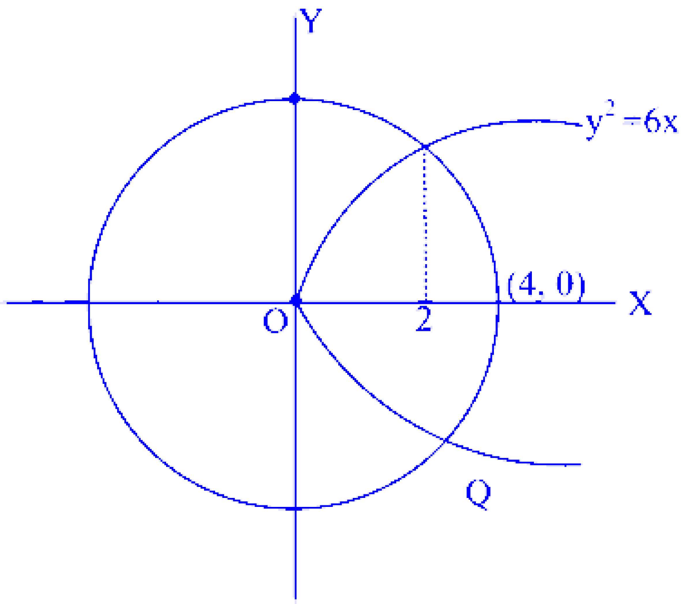
We want the area inside the circle and outside the parabola, between $x=0$ and $x=4$.

To get this area, we split it into two parts:

- From $x=0$ to $x=2$ (where the parabola is below the circle), the top curve is the parabola: $y=\sqrt{6x}$
- From $x=2$ to $x=4$ (where only the circle remains), the top curve is the circle: $y=\sqrt{16-x^2}$

So, the area is:

$$\text{Area}=2\left[\int_0^2 y_{\text{parabola}} \quad dx+\int_2^4 y_{\text{circle}} \quad dx\right]$$



So,$=2\int_0^2\sqrt{6x}\,dx+2\int_2^4\sqrt{16-x^2}\,dx$

Solving the Integrals:

First, $\int_0^2\sqrt{6x}\,dx$:

$\sqrt{6x}=\sqrt{6}\sqrt{x}$, so

$$\int_0^2\sqrt{6x}dx=\sqrt{6}\int_0^2 x^{1/2}dx=\sqrt{6}\cdot\left[\frac{2}{3}x^{3/2}\right]_0^2=\sqrt{6}\cdot\frac{2}{3}\left[(2)^{3/2}-0\right]$$

Second, $\int_2^4\sqrt{16-x^2}\,dx$:

The formula for this is:

$\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right)$

Here $a = 4$. Plug in $x = 4$ and $x = 2$ to get the answer.

Now, combine both solutions:

$$
\begin{aligned}
&= 2\sqrt{6} \cdot \frac{2}{3}\left(2^{3/2}\right) + 2\left[\frac{x}{2}\sqrt{16 - x^2} + 8\sin^{-1}\left(\frac{x}{4}\right)\Big|_{x=2}^{x=4}\right] \\
&= \frac{4}{3}\sqrt{6} \cdot 2\sqrt{2} + 2\left[\left(0 + 8\frac{\pi}{2}\right) - \left(2\sqrt{3} + 8\frac{\pi}{6}\right)\right] \\
&= \frac{16\sqrt{3}}{3} + 2\left(\frac{8\pi}{3} - 2\sqrt{3}\right) \\
&= \frac{16\sqrt{3}}{3} + \frac{16\pi}{3} - 4\sqrt{3} \\
&= \frac{4\sqrt{3}}{3} + \frac{16\pi}{3} \\
&= \frac{4}{3}(4\pi + \sqrt{3})
\end{aligned}
$$

Question2

The area (in sq. units) of the region bounded by the curves $y = x^2$ and $y = 8 - x^2$ is

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Options:

A.

$\frac{32}{3}$

B.

$\frac{16}{3}$

C.

$\frac{64}{3}$

D.

$\frac{128}{3}$

Answer: C

Solution:

Given, curves are $y=x^2$ and

$y=8-x^2$

Equating these two equations, we get

$x^2=8-x^2 \Rightarrow 2x^2=8$

$\Rightarrow \quad x^2=4 \Rightarrow x=\pm 2$

$$\begin{aligned} \text{Thus, area } &= \int_{-2}^{2}\left[\left(8-x^2\right)-x^2\right]dx \\ &= \int_{-2}^{2}\left(8-x^2-x^2\right)dx=\int_{-2}^{2}\left(8-2x^2\right)dx \\ &= \left[8x-\tfrac{2}{3}x^3\right]_{-2}^{2} \\ &= \left\{8(2)-\tfrac{2}{3}(2)^3\right\}-\left\{8(-2)-\tfrac{2}{3}(-2)^3\right\} \\ &= \left(16-\tfrac{16}{3}\right)-\left(-16+\tfrac{16}{3}\right) \\ &= 16-\tfrac{16}{3}+16-\tfrac{16}{3} \\ &= 32-\tfrac{32}{3}=\tfrac{96-32}{3}=\tfrac{64}{3} \end{aligned}$$

$\therefore$ The area of the region bounded by the curves is $\frac{64}{3}$ sq. units.

Question3

Area of the region (in sq. units) bounded by the curve $y=x^2-5x+4, x=0, x=2$ and the X-axis is

AP EAPCET 2025 - 23rd May Evening Shift

Options:

A.

$\frac{8}{3}$

B.

3

C.

5

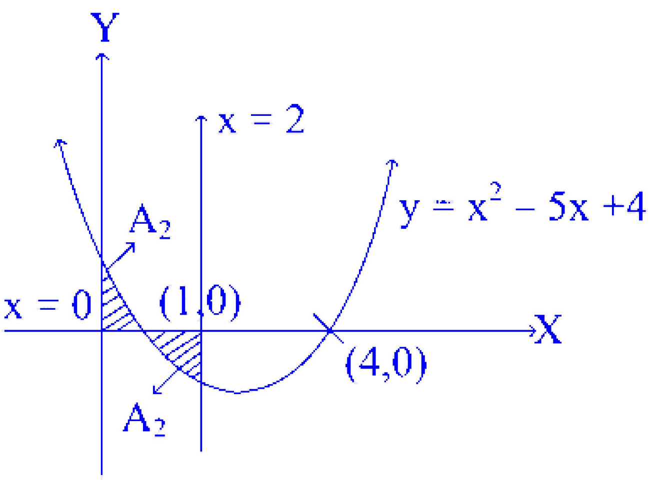
D.

$\frac{5}{2}$

Answer: B

Solution:

$y=x^2-5x+4=(x-1)(x-4)$



$$A=A_1+A_2=\int_0^1\left(x^2-5x+4\right)dx+\int_1^2-\left(x^2-5x+4\right)$$
$$=\left[\frac{x^3}{3}-\frac{5x^2}{2}+4x\right]_0^1+\left[\frac{-x^3}{3}+\frac{5x^2}{2}-4x\right]_1^2$$
$$=\left(\frac{1}{3}-\frac{5}{2}+4\right)+\left(-\frac{8}{3}+10-8+\frac{1}{3}-\frac{5}{2}+4\right)$$
$$=\left(\frac{3}{2}+\frac{1}{3}\right)+\left(6-\frac{7}{3}-\frac{5}{2}\right)$$
$$=\frac{9+2}{6}+\left(\frac{36-14-15}{6}\right)$$
$$=\frac{11}{6}+\frac{7}{6}=\frac{18}{6}=3$$

Question4

The area (in sq. units) of the region bounded by the lines $x=0, x=\frac{\pi}{2}$ and $f(x)=\sin x, g(x)=\cos x$ is

AP EAPCET 2025 - 23rd May Morning Shift

Options:

A.

$2(\sqrt{2}-1)$

B.

$2(\sqrt{2}+1)$

C.

$2(\sqrt{3}-1)$

D.

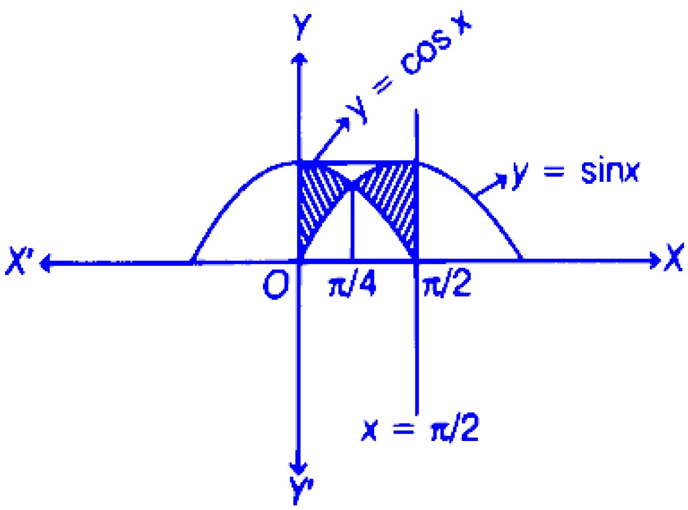
$3\sqrt{2}+1$

Answer: A

Solution:

Required area (A)

$$= \int_0^{\frac{\pi}{4}} (\cos x - \sin x) dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x) dx$$



$$=(\sin x + \cos x)_0^{\frac{\pi}{4}} + (-\cos x - \sin x)_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$
$$=\left[\left(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}\right) - (\sin 0 + \cos 0)\right] - \left[\left(\cos \frac{\pi}{2} + \sin \frac{\pi}{2}\right) - \left(\cos \frac{\pi}{4} + \sin \frac{\pi}{4}\right)\right]$$
$$=(\sqrt{2}-1) - (1-\sqrt{2})$$
$$=\sqrt{2}-1-1+\sqrt{2}$$
$$=2\sqrt{2}-2 = 2(\sqrt{2}-1) \text{ sq. unit}$$

Question5

The area of the region lying between the curves $y=\sqrt{4-x^2}, y^2=3x$ and the Y-axis is

AP EAPCET 2025 - 22nd May Morning Shift

Options:

A.

$\frac{\pi}{3}-\frac{1}{2\sqrt{3}}$

B.

$\frac{\pi}{6}+\frac{1}{2\sqrt{3}}$

C.

$\frac{\pi}{3}+\frac{1}{2\sqrt{3}}$

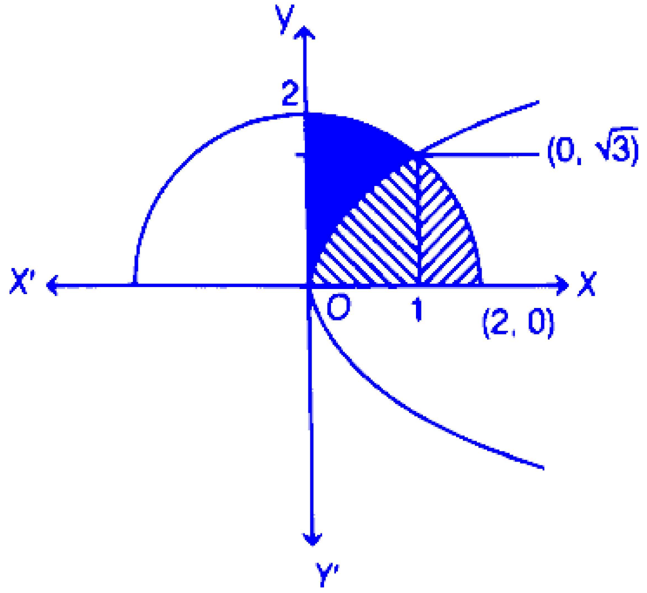
D.

$\frac{\pi}{6}-\frac{1}{2\sqrt{3}}$

Answer: A

Solution:

$y=\sqrt{4-x^2}$ and $y^2=3x$



$\Rightarrow \quad 3x = 4 - x^2$

$\Rightarrow \quad x^2 + 3x - 4 = 0$

$\Rightarrow \quad (x + 4)(x - 1) = 0$

$\quad x = 1, -4 \Rightarrow y = \sqrt{3}$

$\therefore$ Required area,

$$= \frac{1}{3}\int_0^{\sqrt{3}} y^2 dy + \int_{\sqrt{3}}^{2} \sqrt{4 - y^2} dy$$

$$= \frac{1}{3}\left[\frac{y^3}{3}\right]_0^{\sqrt{3}} + \left[\frac{y}{2}\sqrt{4 - y^2} + \frac{4}{2}\sin^{-1}\frac{y}{2}\right]_{\sqrt{3}}^{2}$$

$$= \frac{1}{9}(3\sqrt{3}) + 2\frac{\pi}{2} - \frac{\sqrt{3}}{2} - 2\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{3}}{3} - \frac{\sqrt{3}}{2} + \frac{\pi}{3} = \frac{\pi}{3} - \frac{1}{2\sqrt{3}}$$

Question6

The area of the region (in sq. units) enclosed between the curves $y = |x|, y = [x]$ and the ordinates $x = -1, x = 0, x = 1$ is

AP EAPCET 2025 - 21st May Evening Shift

Options:

A.

2

B.

3/2

C.

3

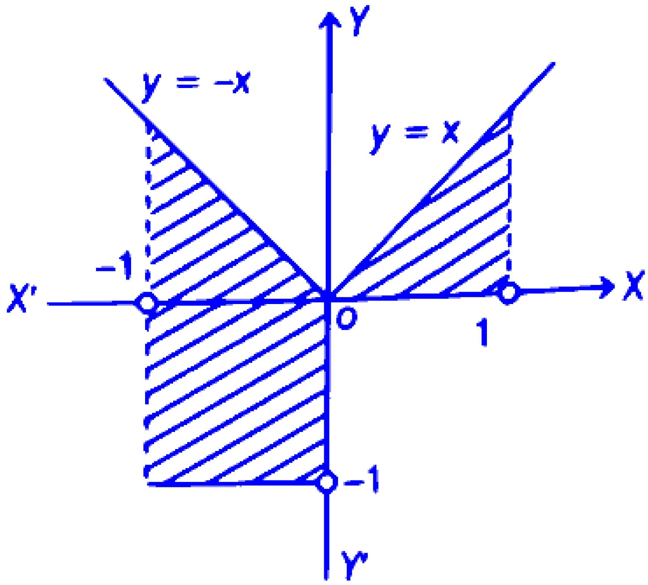
D.

5/2

Answer: A

Solution:

Area of enclosed region,



$$= 2\left(\frac{1}{2} \times 1 \times 1\right) + (x)$$
$$= 1 + 1 = 2$$

Question7

If (a, β) is the stationary point of the curve $y = 2x - x^2$, then the area bounded by the curves $y = 2^x, y = 2x - x^2, x = 0$ and $x = \alpha$ is

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Options:

A. $\frac{3\log 2+4}{2}$

B. $\frac{3+\log 4}{6}$

C. $\frac{3-\log 4}{3\log 2}$

D. $\frac{1}{\log 2}+\frac{3}{4}$

Answer: C

Solution:

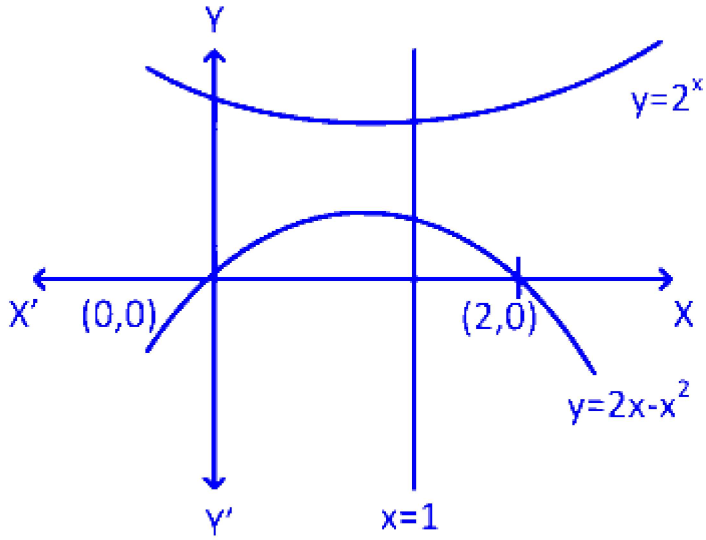
As (α, β) is the stationary point of

$$y = 2x - x^2, \frac{dy}{dx} = 0$$

$$2 - 2x$$

$$\Rightarrow \quad x = 0$$

$$\Rightarrow \quad \alpha = 1$$



$\therefore$ Required area $= \int_0^1 (y_2 - y_1)dx$

$$= \int_0^1 \left(2^x - 2x + x^2\right)\Big) dx = \left[\frac{2^x}{\log 2} - x^2 + \frac{x^3}{3}\right]_0^1$$

$$= \left[\frac{2^1}{\log 2} - 1 + \frac{1}{3} - \frac{1}{\log 2}\right]$$

$$= \frac{2}{\log 2} - 1 + \frac{1}{3} - \frac{1}{\log 2}$$
$$= \frac{1}{\log 2} - \frac{2}{3}$$
$$\Rightarrow \frac{3 - \log 4}{3 \log 2}$$

Question8

The area (in sq units) bounded by the curves $x = y^2$ and $x = 3 - 2y^2$ is

AP EAPCET 2024 - 22th May Morning Shift

Options:

A. 8

B. $8/3$

C. 4

D. 6

Answer: C

Solution:

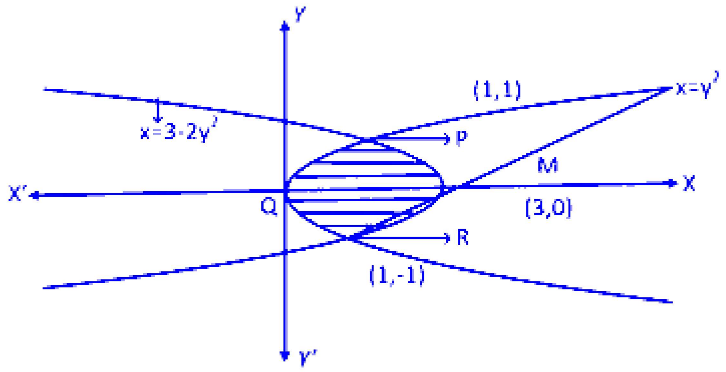
$$x = y^2, x = 3 - 2y^2$$
$$y = \sqrt{x}$$
$$y = \sqrt{\frac{3 - x}{2}}$$

Vertices $(0, 0)$ and $(3, 0)$

intersect at (1,1) and (1, -1)

Area of shaded region

$= 2 \times$ (Area of PQM)



$$= 2\left\{\left[\int_0^1 \sqrt{x}dx\right] + \left[\int_1^3 \sqrt{\frac{3-x}{2}}dx\right]\right\}$$

$$= 2\left[\left(\frac{2}{3}x^{3/2}\right)_0^1 + \left(\frac{-1}{\sqrt{2}} \cdot \frac{2}{3}(3-x)^{3/2}\right)\right]_1^3$$

$$= 2\left[\frac{2}{3} - \left(0 - \frac{1}{\sqrt{2}} \cdot \frac{2}{3}2^{3/2}\right)\right]$$

$$= 2\left(\frac{2}{3} + \frac{4}{3}\right) = 2 \times \frac{6}{3} = 4$$

Area of required region $= 4$sq units

Question9

Area of the region enclosed by the curves $3x^2 - y^2 - 2xy + 4x + 1 = 0$ and $3x^2 - y^2 - 2xy + 6x + 2y = 0$ is

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Options:

A. $\frac{3}{4}$

B. $\frac{1}{4}$

C. 1

D. $\frac{1}{2}$

Answer: B

Solution:

Given curves are

$$3x^2 - y^2 - 2xy + 4x + 1 = 0 \qquad \ldots(i)$$
$$\text{and } 3x^2 - y^2 - 2xy + 6x + 2y = 0 \qquad \ldots(ii)$$

Eqs. (i) and (ii) can be written as

$(3x + y + 1)(x - y + 1) = 0$, and $(3x + y)(x - y + 2) = 0$, respectively.

Thus, the curve (i) represents the pair of lines

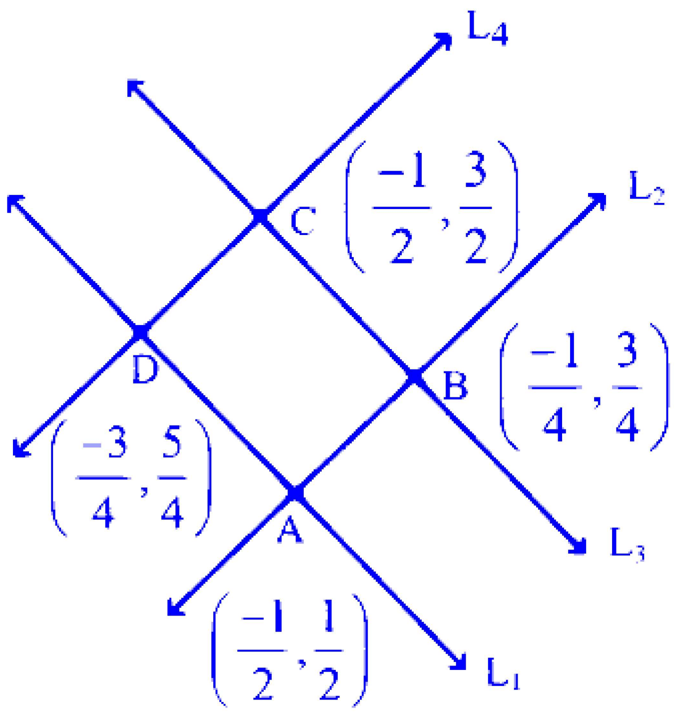
$$L_1 : 3x + y + 1 = 0$$
$$L_2 : x - y + 1 = 0$$

and the curve (ii) represents the pair of lines

$$L_3 : 3x + y = 0$$
$$L_4 : x - y + 2 = 0$$

Here, $L_1 \| L_3$ and $L_2 \| L_4$

So, the region closed by the given curves will be a parallelogram.



Now, required area $=$ Area of parallelogram $ABCD = 2 \times$ (area of $\triangle ABC$)

$$= 2 \times \frac{1}{2} \begin{vmatrix} -\frac{1}{2} & \frac{1}{2} & 1 \\ -\frac{1}{4} & \frac{3}{4} & 1 \\ -\frac{1}{2} & \frac{3}{2} & 1 \end{vmatrix}$$
$$= \left[-\frac{1}{2}\left(\frac{3}{4} - \frac{3}{2}\right) - \frac{1}{2}\left(-\frac{1}{4} + \frac{1}{2}\right) + 1\left(-\frac{3}{8} + \frac{3}{8}\right)\right]$$
$$= -\frac{1}{2}\left(-\frac{3}{4}\right) - \frac{1}{2}\left(\frac{1}{4}\right) + 0$$
$$= \frac{3}{8} - \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

Question10

The area of the region under the curve $y = |\sin - \cos x|, 0 \leq x \leq \frac{n}{2}$ and above X-axis, is (in sq units)

AP EAPCET 2024 - 21th May Morning Shift

Options:

A. $2\sqrt{2}$

B. $2\sqrt{2} - 1$

C. $2(\sqrt{2} - 1)$

D. $2(\sqrt{2} + 1)$

Answer: C

Solution:

To find the area under the curve $y = |\sin x - \cos x|$ from $x = 0$ to $x = \frac{\pi}{2}$, and above the X-axis, we proceed as follows:

The expression $|\sin x - \cos x|$ changes based on x. Solving $|\sin x - \cos x| = 0$:

$$\begin{aligned} \sin x - \cos x &= 0 \\ \sin x &= \cos x \\ \tan x &= 1 \\ x &= \frac{\pi}{4} \end{aligned}$$

This implies that $\sin x - \cos x$ is negative from 0 to $\frac{\pi}{4}$ and positive from $\frac{\pi}{4}$ to $\frac{\pi}{2}$.

Thus, the function can be defined piecewise as:

$$|\sin x - \cos x| = \begin{cases} \cos x - \sin x & \text{for } 0 \leq x < \frac{\pi}{4} \\ \sin x - \cos x & \text{for } \frac{\pi}{4} \leq x \leq \frac{\pi}{2} \end{cases}$$

Integrating from $x = 0$ to $x = \frac{\pi}{4}$:

$$\int_0^{\pi/4} (\cos x - \sin x)\, dx = \int_0^{\pi/4} \cos x\, dx - \int_0^{\pi/4} \sin x\, dx$$

Calculating each integral:

$$\int_0^{\pi/4} \cos x\, dx = [\sin x]_0^{\pi/4} = \sin \frac{\pi}{4} - \sin 0 = \frac{\sqrt{2}}{2}$$

$\int_0^{\pi/4} \sin x \, dx = [-\cos x]_0^{\pi/4} = -\cos\frac{\pi}{4} + \cos 0 = -\frac{\sqrt{2}}{2} + 1$

Thus, the area from $x = 0$ to $x = \frac{\pi}{4}$ is:

$\int_0^{\pi/4} (\cos x - \sin x) \, dx = \frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} + 1\right) = \sqrt{2} - 1$

Integrating from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{2}$:

$\int_{\pi/4}^{\pi/2} (\sin x - \cos x) \, dx = \int_{\pi/4}^{\pi/2} \sin x \, dx - \int_{\pi/4}^{\pi/2} \cos x \, dx$

Calculating each integral:

$\int_{\pi/4}^{\pi/2} \sin x \, dx = [-\cos x]_{\pi/4}^{\pi/2} = -\cos\frac{\pi}{2} + \cos\frac{\pi}{4} = 0 + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

$\int_{\pi/4}^{\pi/2} \cos x \, dx = [\sin x]_{\pi/4}^{\pi/2} = \sin\frac{\pi}{2} - \sin\frac{\pi}{4} = 1 - \frac{\sqrt{2}}{2}$

Thus, the area from $x = \frac{\pi}{4}$ to $x = \frac{\pi}{2}$ is:

$\int_{\pi/4}^{\pi/2} (\sin x - \cos x) \, dx = \frac{\sqrt{2}}{2} - \left(1 - \frac{\sqrt{2}}{2}\right) = \sqrt{2} - 1$

Total Area:

Adding the two parts:

$(\sqrt{2} - 1) + (\sqrt{2} - 1) = 2(\sqrt{2} - 1)$

Thus, the total area under the curve is $2(\sqrt{2} - 1)$ square units.

Question11

Area of the region (in sq units) enclosed by the curves $y^2 = 8(x + 2), y^2 = 4(1 - x)$ and the Y-axis is

AP EAPCET 2024 - 20th May Morning Shift

Options:

A. $\frac{8}{3}(5 - 3\sqrt{2})$

B. $\frac{8}{3}(\sqrt{2} - 1)$

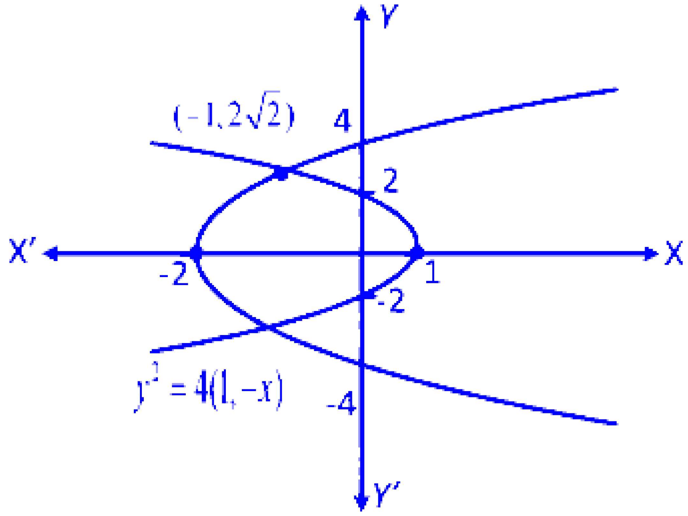
C. $\frac{8}{3}(3 - \sqrt{2})$

D. $\frac{4}{3}(\sqrt{2} + 1)$

Answer: A

Solution:

We have,



$y^2 = 8(x+2)$ and $y^2 = 4(1-x)$

The intersecting point of the above 2 parabola are $(-1, \pm 2\sqrt{2})$

$\therefore$ Required area

$$= \left| 2\left[\int_2^{2\sqrt{2}} \left(1 - \frac{y^2}{4}\right) dy + \int_{2\sqrt{2}}^{4} \left(\frac{y^2}{8} - 2\right) dy\right]\right.$$

$$= \left| 2\left[\left[y - \frac{y^3}{12}\right]_2^{2\sqrt{2}} + \left[\frac{y^3}{24} - 2y\right]_{2\sqrt{2}}^{4}\right]\right.$$

$$= \left| 2\left[\left(2\sqrt{2} - \frac{16\sqrt{2}}{12}\right) - \left(2 - \frac{8}{12}\right) + \left(\frac{64}{24} - 8\right)\right.\right.$$

$$\left. - \left(\frac{16\sqrt{2}}{24} - 4\sqrt{2}\right)\right]$$

$$= \left| 2\left[\frac{2\sqrt{2}}{3} - \frac{4}{3} - \frac{16}{3} + \frac{10\sqrt{2}}{3}\right]\right.$$

$$= \left| 2\left[\frac{12\sqrt{2}}{3} - \frac{20}{3}\right]\right|$$

$$= \left|\frac{8}{3}[3\sqrt{2} - 5]\right| = \frac{8}{3}(5 - 3\sqrt{2})$$

Question12

The area (in sq units) of the smaller region lying above the X-axis and bounded between the circle $x^2+y^2=2ax$ and the parabola $y^2=ax$ is

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Options:

A. $2a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$

B. $a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$

C. $a^2\left(\frac{\pi}{4}+\frac{2}{3}\right)$

D. $a^2\left(\frac{\pi^2}{4}-\frac{1}{3}\right)$

Answer: B

Solution:

Given the problem of finding the area of a region bounded by a circle and a parabola, let's explore how to calculate it.

The equation of the circle is:

$x^2+y^2=2ax$

This can be rewritten by completing the square for x:

$(x-a)^2+y^2=a^2$

The equation of the parabola is:

$y^2=ax$

Finding Points of Intersection:

To find the intersection points between the circle and the parabola, we set:

$x^2+ax=2ax$

$x^2-ax=0$

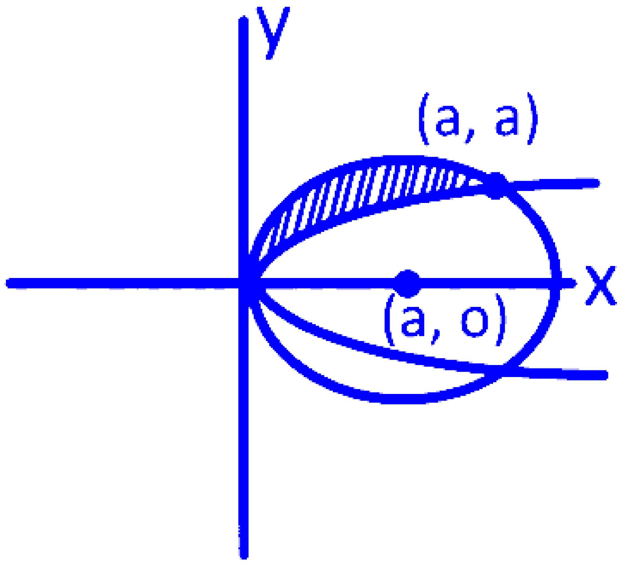
$x(x-a)=0$

Thus, $x=0$ or $x=a$.

For $x=0$, substituting in the parabola $y^2=ax$, we get $y=0$.

For $x=a$, substituting in the parabola $y^2=ax$, we get $y=\pm a$.

Thus, the points of intersection are $(0,0)$, (a,a), and $(a,-a)$.



Calculating the Area:

We need to integrate the area between the circle and the parabola from $x=0$ to $x=a$:

Required Area $=\int_0^a\left(\sqrt{a^2-(x-a)^2}-\sqrt{a}\sqrt{x}\right) dx$

This evaluates to:

$=\left[\frac{x-a}{2}\sqrt{a^2-(x-a)^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)-\sqrt{a}\times\frac{2}{3}x^{\frac{3}{2}}\right]_0^a$

$=\left[-\sqrt{a}\cdot a^{\frac{3}{2}}\times\frac{2}{3}-\frac{a^2}{2}\cdot\sin^{-1}(-1)\right]$

$=-\frac{2}{3}a^2+\frac{a^2}{2}\times\frac{\pi}{2}$

Therefore, the area of the smaller region lying above the X-axis is:

$a^2\left(\frac{\pi}{4}-\frac{2}{3}\right)$

Question13

The area of the region (in sq units) enclosed by the curve $y=x^3-19x+30$ and the X-axis, is

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Options:

A. $\frac{167}{2}$

B. $\frac{517}{2}$

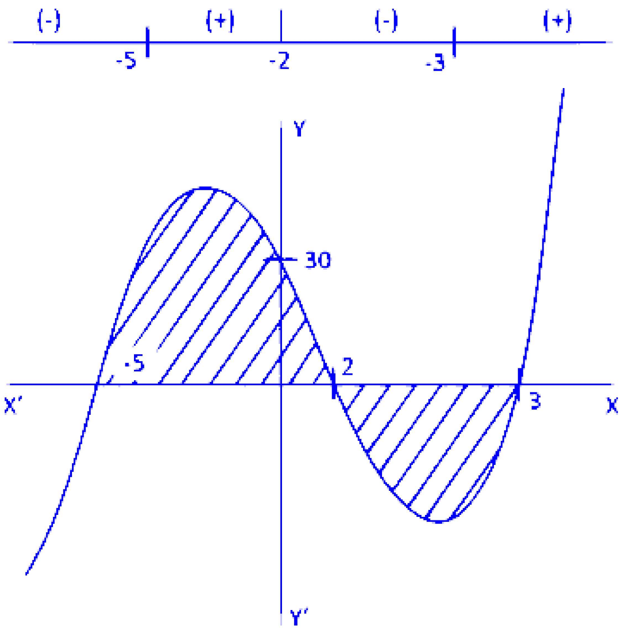
C. 36

D. 72

Answer: B

Solution:

$$(y) = x^3 - 19x + 30$$
$$= (x-2)(x-3)(x+5)$$



$\therefore$ Required area

$$= \int_{-5}^{2}\left(x^3 - 19x + 30\right)dx + \left|\int_{2}^{3} x^3 - 19x + 30dx\right|$$

$$= \left[\frac{x^4}{4} - \frac{19x^2}{2} + 30x\right]_{-5}^{2}$$

$$+ \left|\left[\frac{x^4}{4} - \frac{19x^2}{2} + 30x\right]_{2}^{3}\right|$$

$$= (4 - 38 + 60) - \left(\frac{625}{4} - \frac{475}{2} - 150\right)$$

$$+ \left|\left(\frac{81}{4} - \frac{171}{2} + 90\right) - (4 - 38 + 60)\right|$$

$$= 26 + \frac{925}{4} + \left|\frac{99}{4} - 26\right| = \frac{1029}{4} + \frac{5}{4}$$

$$= \frac{1034}{4} = \frac{517}{2}$$
